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On the phase diagram of the Josephson junction

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Abstract. The phase diagram of an extended Josephson junction in the magnetic field–temperature plane is constructed. It contains a floating (incommensurate) phase, which corresponds to the usual intermediate state, high-order commensurate phases and a liquid-crystal-like disordered phase. It is shown that the disappearance of the Josephson effect is a weak first-order phase transition connected to bulk superconductivity breaking. The influence of defects is also considered.

1. Introduction

It is widely known that a Josephson junction placed in a longitudinal magnetic field is the two-dimensional analogue of a type-two superconductor. The 'classic' papers here are Kulik (1966) and Owen and Scalapino (1967). Subsequently, this approach has been developed to the point, for example, of being employed in the design and experimental testing of practical Josephson elements for computer applications. This body of work and recent theoretical developments are reviewed by Barone and Paterno (1982).

If the external field exceeds some critical value H_{c1} the Meissner state becomes unstable with respect to spontaneous arising of Josephson vortices. It follows from the electrodynamics of the weak superconductivity that the phase transition into the intermediate state is continuous. It means that the Josephson vortices repulse each other provided the distance between them is large. The picture outlined is similar to the commensurate–incommensurate (IC) phase transition (Bak 1982). It is easy to check by comparing corresponding equations that this similarity is very deep. It is interesting to note that the authors of the latest reviews devoted to IC phases (Fisher 1984, 1986, Nattermann and Villain 1988) did not consider the Josephson junction as a system with an IC phase‡. The only work that uses this analogy (Browne and Horovitz 1988) is devoted to the interpretation of the I–V characteristics of high-T_c superconductors near the bulk phase transition (Dubson *et al* 1988, Stamp *et al* 1988). The suggestions of Browne and Horovitz (1988) that related to the vicinity of T_c will be discussed below. However, there are many interesting phenomena that should take place in all the region of existence of weak superconductivity. The aim of the present work is to fill in this gap.

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‡ This corresponds to the history of the discovery of IC phases (Bak 1982). The theory has been rediscovered independently, in widely different contexts, many times. That is why it is necessary to include Kulik in the list of authors of this seminal discovery.

We shall construct the phase diagram of the Josephson junction at finite temperature, and study the discreteness effects and the influence of weak disorder. It is quite probable that the Josephson junction as a real two-dimensional system will represent more possibilities for experimental verification of the very interesting theoretical predictions than the usual objects (Fisher 1986). The results obtained have a general character, but the most prominent effects should be observed in high-T_c materials.

2. Phase transition into the intermediate state at finite temperature

First, let us remember how the phase transition into the vortex state takes place at zero temperature (Kulik 1966, Owen and Scalapino 1967, Kulik and Yanson 1970, Barone and Paterno 1982). The free energy per unit length of a Josephson vortex in an external field H has the form

$$\varepsilon - H\Phi_0/4\pi$$

where ε is the linear tension of a vortex and Φ_0 is the flux quantum. If we account for the inter-vortex repulsion, which depends on their mean distance $1/n$,

$$\mathrm{const}(\Phi_0^2/\lambda\delta)\exp(-1/n\delta)$$

(here λ and δ are the London and Josephson penetration depths, respectively: to be specific we consider the Josephson junction consisting of the same superconductors and the thickness of the junction is small compared to λ) and we multiply the resulting relation by the vortex density n, we obtain the formula for the free energy per unit area:

$$g = n[\varepsilon - H\Phi_0/4\pi + \mathrm{const}(\Phi_0^2/\lambda\delta)\exp(-1/n\delta)]. \tag{1}$$

Equation (1) makes sense provided that $n\delta \ll 1$. If the external field is less than $H_{c1} = 4\pi\varepsilon/\Phi_0$ the minimization of g with respect to n leads to the conclusion $n = 0$, which means stability of the Meissner state. For $H > H_{c1}$ the free energy of a vortex becomes negative and vortices begin to arise spontaneously. However, owing to repulsion among them (the second term on the RHS of (1)), the vortex density increases continuously from zero. Minimization with respect to n gives

$$n\delta \simeq 1/\ln[H_{c1}/(H - H_{c1})] \tag{2}$$

where $H_{c1} = \Phi_0/\lambda\delta$. Magnetic induction B is connected to the vortex density by relation $B = n\Phi_0/\lambda$. Therefore, for magnetization M near H_{c1} we obtain (Kulik 1966)

$$(\mathrm{d}M/\mathrm{d}H)|_{H\to H_{c1}+0} \simeq H_{c1}/(H - H_{c1})\ln^2[H_{c1}/(H - H_{c1})]. \tag{3}$$

Let us account for thermal fluctuations by means of the methods described by Fisher (1986) and Nattermann and Villain (1988). The elastic energy of a vortex has the form

$$H = \int \mathrm{d}y\, \varepsilon(\mathrm{d}x/\mathrm{d}y)^2/2. \tag{4}$$

We assume that magnetic field is directed along the y direction and x indicates the displacement of a vortex from a reference line. The main difference between the system under consideration and earlier studied two-dimensional ones (Fisher 1984) is the absence of the singularity of linear tension ε at $T \to 0$.

The free-energy contribution due to thermal fluctuations can be estimated from equation (4) by averaging the integrand taken with opposite sign (since thermal fluctuations diminish the free energy):

$$f = \varepsilon - \text{const}\,\varepsilon\langle(\mathrm{d}x/\mathrm{d}y)^2\rangle = \varepsilon - \text{const}\,\varepsilon \int \frac{\mathrm{d}k}{2\pi} k^2\langle|x_k|^2\rangle$$

$$= \varepsilon - \text{const} \int_{1/L}^{1/\delta} \mathrm{d}k = \varepsilon - \text{const}\, T/\delta + \text{const}\, T/L. \quad (5)$$

The lower bound in the integral is the inverted vortex length L^{-1} and the upper one is the inverted Josephson penetration depth δ^{-1} (since the maximal frequency of the Josephson vortex oscillation, the so-called Josephson plasma frequency ω_J, corresponds to wavenumber $k \simeq \delta^{-1}$ (Kulik 1966, Kulik and Yanson 1970)). The second term in equation (5) is the entropic contribution to the vortex free energy and the third one is the finite-size correction. For a single flux line the last one goes to zero at $L \to \infty$; however, it leads to a specific entropic repulsion if we consider a chain of vortices. Let us calculate the RMS deviation $w(L) = \{\langle[x(L) - x(0)]^2\rangle\}^{1/2}$. From the Hamiltonian (4) it follows that

$$w(L) \simeq (TL/\varepsilon)^{1/2}. \quad (6)$$

The neighbouring vortices restrict maximal transverse deviation of a given vortex due to mutual 'collisions'. Hence the maximal deviation is equal approximately to $1/n$ and the 'free collision length' is equal to

$$L_{\max} \simeq \varepsilon/Tn^2. \quad (7)$$

The substitution of $L_{\max}$ into the third term in (5) leads to the expression for the free energy at finite temperature:

$$g = n[\varepsilon - H\Phi_0/4\pi - \text{const}\, T/\delta + \text{const}(\Phi_0^2/\lambda\delta)\exp(-1/n\delta) + \pi^2T^2n^2/6\varepsilon]. \quad (8)$$

The numerical factor in the third term in (8) can be recovered on comparison with the exact solution of a similar problem (Pokrovsky and Talapov 1979). Equating the first term on the RHS of the last relation to zero, we obtain the temperature dependence of the lower critical field

$$H_{c1}(T) = H_{c1}(0)(1 - \text{const}\, T/\varepsilon\delta). \quad (9)$$

Formulae (8) and (9) make sense if the quantum effects are negligible. The corresponding condition has the form

$$T \gg \hbar\omega_J \quad (10)$$

where $\hbar$ is the Planck constant. The above condition really means that the temperature should exceed several degrees (Kulik and Yanson 1970). The third term in (8) dominates the second one for small n. Upon minimization of (8) with respect to n at $n \to 0$, one obtains the seminal result of Pokrovsky and Talapov (1979):

$$n = (\Phi_0\varepsilon/2\pi^2T^2)^{1/2}(H - H_{c1})^{1/2}. \quad (11)$$

Correspondingly, instead of equation (3) we have

$$(\mathrm{d}M/\mathrm{d}H|_{H\to H_{c1}} \simeq (\Phi_0^3\varepsilon/T^2\lambda^2)(H - H_{c1})^{-1/2}. \quad (12)$$

Formulae (11) and (12) are asymptotically exact in the vicinity of H_{c1}. The width of this region can be estimated by comparing equations (2) and (11):

$$H_m - H_{c1} \simeq T^2/\delta^2\Phi_0\varepsilon \ln^2(\varepsilon\delta/T). \quad (13)$$

One has to use equation (12) with care because it will be shown below that inside the interval (13) the Josephson flux lattice is melted and it is, in fact, similar to a two-dimensional liquid crystal.

3. On the long-range order in the intermediate state

The flux line chain arising at $H > H_{c1}$ can be considered as a two-dimensional crystal. Its elastic constants were, in fact, calculated by Lyuksyutov and Pokrovsky (1982). The compression modulus K_1 of the vortex lines can be obtained from expression (8) for the free energy:

$$K_1 = n^2(\mathrm{d}^2g/\mathrm{d}n^2) \simeq (C\Phi_0^2/\lambda\delta^3 n)\exp(-1/n\delta) + \pi^2T^2n^3/\varepsilon. \tag{14}$$

Here C is a numerical constant of the order of unity. The tilt modulus K_2 can be associated with the tension of a vortex

$$K_2 = \varepsilon n. \tag{15}$$

The elastic energy of a vortex chain can be written as follows:

$$U = \tfrac{1}{2}\int \mathrm{d}x\,\mathrm{d}y[K_1(\partial u/\partial x)^2 + K_2(\partial u/\partial y)^2] \tag{16}$$

where u is the transverse displacement of a vortex. It is known due to Peierls and Landau (Landau and Lifshitz 1976) that long-range translational order is absent in a two-dimensional crystal that is described by the Hamiltonian (16). For the intermediate state of the Josephson junction this result was first obtained by Fetter and Stephen (1968). Using equation (16) it is easy to show that the RMS deviation of the vortex lattice increases as the logarithm of the system size. Therefore, instead of the IC phase one obtains a 'floating' phase with no long-range order but with an algebraic decay of correlation functions (Bak 1982). Note that the thermal fluctuations rather than the quantum ones destroy the long-range order. The energy U can be reduced to the Hamiltonian of the XY model. It is known that melting of the floating phase via the Berezinsky–Kosterlitz–Thouless (BKT) dislocation-mediated mechanism is possible in this case (Kosterlitz and Thouless 1973). Such melting is quite probable near H_{c1} where elastic moduli K_1 and K_2 are small. Using directly the equation for the transition temperature of the Kosterlitz–Thouless theory (Lyuksyutov and Pokrovsky 1982) one obtains

$$T_m = (K_1K_2)^{1/2}b^2/8\pi \tag{17}$$

where b is the Burgers dislocation vector. In the case under consideration $b = 1/n$. Inserting (14) and (15) into (17) one finds the equation

$$\delta n \simeq 1/\ln(\Phi_0^2/T_m\lambda) \simeq 1/\ln(\varepsilon\delta/T_m). \tag{18}$$

It is clear that on the melting curve (17) and (18) the second and the third terms in equation (8) are equal within an order of magnitude. Therefore, the expression (13) is the melting curve also. Hence, at fields slightly above H_{c1} the vortex structure has not even algebraic order and it is equivalent to a two-dimensional liquid crystal (since the orientational order persists). Inside this region the combination of elastic constants $(K_1K_2)^{1/2}$ is renormalized by the free dislocations to zero. If the external field achieves H_m (13), the BKT phase transition takes place and the floating phase with algebraic order arises. The combination of elastic moduli $(K_1K_2)^{1/2}$ increases discontinuously from zero to some finite value at $H = H_m$; above the transition it has the dependence on T or H specific to the BKT theory. Inside the floating phase K_1 and K_2 are expressed by formulae (14) and (15). The nature of the phase transition at $H = H_{c1}$ and the form of the magnetization curve between H_{cl} (9) and H_m (13) are unknown. But near the boundary between liquid crystal and floating phase the magnetization curve has to be similar to (12).

Up to now we have neglected discreteness effects. They would be important at low temperatures since the divergence of the RMS deviation of a vortex lattice is very weak (the logarithmic one). If the crystalline potential suppresses the fluctuations we have in fact a high-order commensurate phase with the usual long-range order. The organization of such phases is described by Uimin and Pokrovsky (1984). All the phases can be divided in two groups: the simple and the complex ones. The first group consists of structures that have a period different from the period of the neighbouring simple ones by the crystalline lattice spacing. The complex structures are constructed from an arbitrary sequence of neighbouring simple ones. If the temperature is equal to zero and the magnetic field increases, the system jumps through the infinite sequence of commensurate states (the complete 'devil's staircase' (Bak 1982)). At finite temperature the thermal fluctuations destroy the most complex commensurate phases and we have the incomplete 'devil's staircase'. In order to determine the boundary between the floating phase and the incomplete devil's staircase, it is enough to consider the simple structures only (Villain 1980, Uimin and Pokrovsky 1984). If the period of the crystalline lattice is equal to a, one has to consider the generalization of the Hamiltonian (16):

$$U = \int \mathrm{d}x\,\mathrm{d}y\{\tfrac{1}{2}[K_1(\partial u/\partial x)^2 + K_2(\partial u/\partial y)^2] - V\cos(2\pi u/a)\} \tag{19}$$

where $V \simeq (\Phi_0^2 n/\lambda\delta)\exp(-\delta/a)$ (Villain 1980) is the amplitude of the periodic relief. The Hamiltonian (19) describes roughening phase transition (Villain 1980, Nozieres and Gallet 1987), belonging to the BKT universality class, which corresponds to the depinning of the vortex chain from a crystalline relief (transition into the floating phase). Using the Kosterlitz–Thouless relation for the phase transition temperature we have

$$T_p = 2(K_1K_2)^{1/2}a^2/\pi. \tag{20}$$

Note that this equation has a solution if $na \leqslant 1/2^{1/2}$. In the case of the opposite inequality the commensurate phase persists. It is interesting to compare the melting temperature (17) and the pinning–depinning temperature (20):

$$T_p/T_m = 16a^2n^2. \tag{21}$$

It is evident that near the lower critical field ($n \to 0$) $T_p < T_m$ always. After the substitution of the expressions for elastic moduli in equation (20) one obtains the transition curve ($na \ll 1$)

$$H_p - H_{c1} \simeq T_p^2\delta^2/a^4\varepsilon\Phi_0. \tag{22}$$

Let us remember that the absence of the amplitude of the potential V in equations (20) and (22) is a general property of a roughening transition (Nozieres and Gallet 1987). This amplitude determines the value of the reduced temperature interval $t_p = |T - T_p|/T_p$ where the critical singularities are appreciable:

$$t_p \simeq Vn^{-1}\delta/(K_1K_2)^{1/2}a^2 \simeq (\Phi_0^2/T_p\lambda)\exp(-\delta/a).$$

At $V = 0$ the singularity disappears and equation (22) has no meaning. From the last estimate we can see that in order to provide the best conditions for the observation of the pinning–depinning transition it would be desirable to have $t_p \approx 1$, or

$$\delta \simeq a \ln(\Phi_0^2/T_p\lambda).$$

If we take for concreteness $T_p \simeq 10$ K, $\lambda \simeq 10^4$ Å we obtain $\delta/a \simeq$ 15–20. However, taking into account the characteristic value of $\delta \simeq 10^{-2}$ cm (Barone and Paterno 1982) leads to the conclusion that the discreteness effects are observable only in the very near vicinity of T_p. However, they would be quite prominent if we could make artificial periodicity using a Josephson junction of variable periodic thickness. Another way is to pin the periodic chain of the Abrikosov vortices along the surface of the junction (Aslamazov and Gurovich 1984).

4. Limitations and the vicinity of the phase transition

The free-energy expression (8) permits us also to make some qualitative conclusions about the temperature of disappearance of the Josephson effect. It takes place at

$$T_J \simeq \varepsilon\delta \tag{23}$$

and corresponds to the creation of spontaneous vortex loops inside the junction at zero external field. The last expression can also be obtained via application of the Kosterlitz–Thouless criterion to the Josephson Hamiltonian (Kulik 1966). Relation (23) is in accordance with the scaling hypothesis. Indeed, there is only one intrinsic scale length in the two-dimensional system under consideration, namely the Josephson penetration depth, which plays the role of the correlation length near the phase transition. According to the scaling hypothesis ε, the energy of the linear manifold, should go to zero as T_J/δ and one returns to (23).

Comparison of equations (23) and (18) shows that near T_J the above-mentioned condition $n\delta \ll 1$ breaks down. The temperature dependence of H_{c1} in this region was obtained by Browne and Horovitz (1988). Equation (23) with the help of the determination of ε can be rewritten in the form

$$T_J \simeq \Phi_0^2/\lambda \tag{24}$$

which does not contain the Josephson penetration depth. Substitution of the temperature dependence of λ from the Ginzburg–Landau theory (Lifshitz and Pitaevsky 1978) into equation (24) leads to the conclusion that T_J is very close to the temperature of the bulk phase transition T_c:

$$T_c - T_J \simeq T_c k^{-2} Gi. \tag{25}$$

Here k is the Ginzburg–Landau parameter and Gi is the Ginzburg parameter (Landau and Lifshitz 1976, Lifshitz and Pitaevsky 1978). Interval (25) has to lie outside the fluctuation region because we have used the temperature dependence of λ within the framework of the Ginzburg–Landau theory. This condition is evidently fulfilled for type-one superconductors. However, owing to the vector-potential fluctuations the phase transition into the normal state is a discontinuous one (Halperin *et al* 1974). The 'size' of the first-order transition is

$$\Delta T_1 \simeq T_c k^{-6} Gi. \tag{26}$$

Therefore bulk superconductivity breaking takes place below the temperature T_J (25)

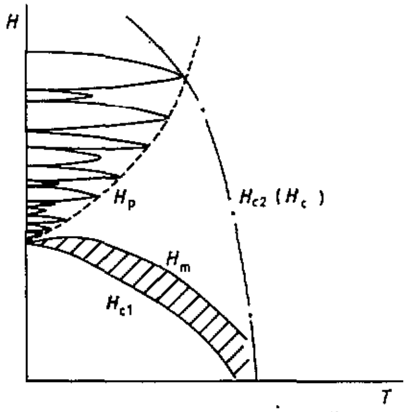

Figure 1. Phase diagram of Josephson junction in magnetic field–temperature plane.

is achieved. Hence the jumps of different quantities which belong to the Josephson effect can be found by substitution of ΔT_1 (26) into the dependences from the Ginzburg–Landau theory.

Let us consider a Josephson junction that consists of type-two superconductors. The analogue of equation (25) follows from the results of Kolomeisky (1988):

$$T_c - T_J \simeq T_c k^{-3} Gi^{3/4}. \tag{27}$$

Note that (27) does not prove the suggestion of Browne and Horovitz (1988) that $T_c > T_J$ since the interval (27) is in fact the boundary of stability of the superconductive state with respect to the spontaneous arising of Abrikosov vortices (Kolomeisky and Levanyuk 1989). Such a bulk transition should be a first-order one and it should take place before the boundary of stability (27) is achieved. Therefore, the disappearance of the Josephson effect is a weak first-order phase transition connected with bulk superconductivity breaking independently of the type of superconductors. The respective temperatures of the first-order transitions are equal.

The results obtained enable us to construct the magnetic field–temperature phase diagram (figure 1). The shaded region between H_m (13) and H_{c1} (9) corresponds to the two-dimensional liquid crystal. The Meissner phase is located at fields below H_{c1}. On the left of H_p (22) one has the IC phase with peninsulas of high-order commensurate phases. The region between H_p and H_m is the floating (IC) phase (the usual intermediate state). The boundary of existence of weak superconductivity is restricted at high fields by the upper critical field (for type-two superconductors) or by the thermodynamic critical field (for type-one superconductors).

Note that even near the phase transition (27) the difference between H_m and H_{c1} is very small: for $T_c \simeq 100$ K it is of order 10^{-4}–10^{-5} Oe. Therefore the experimental discovery of the liquid-crystal phase will be very difficult. However, the observation of the incomplete devil's staircase (on the left of H_p) is probably possible. For example in this region one has to see the magnetization curve with characteristic steps, jumps and continuous parts (Bak 1982). If the temperature increases, the share of the continuous parts will grow and at $H \geqslant H_p$ one obtains the continuous magnetization curve.

5. The influence of defects

Let us consider the influence of weak disorder on the properties of the intermediate state. There are two types of defects from the point of view of a single flux line. The

Abrikosov vortices pinned in the bulk of the superconductors can play the role of random-field-type defects (Aslamazov and Gurovich 1984). The inhomogeneity of the junction thickness can represent the random-bond disorder (for review see Fisher 1986, Nattermann and Villain 1988). The influence of defects is accumulated in a relation like equation (6):

$$w \simeq AL^{\zeta}. \tag{28}$$

Here A depends on both the type and the concentration of defects whereas wandering exponent ζ depends on the type of defects only. The defect-induced wandering of the vortex is qualitatively similar to the above-discussed influence of thermal fluctuations. A similar consideration leads to formulae like equations (11) and (12):

$$n \simeq (A^{-2/\zeta}\Phi_0/\varepsilon)^{\zeta/2(1-\zeta)}(H - H_{c1})^{\zeta/2(1-\zeta)} \tag{29}$$

$$\partial M/\partial H|_{H\to H_{c1}+0} \simeq (\Phi_0/\lambda)(A^{-2/\zeta}\Phi_0/\varepsilon)^{\zeta/2(1-\zeta)}(H - H_{c1})^{(3\zeta-2)/2(1-\zeta)}. \tag{30}$$

For the random bonds $\zeta = 2/3$ (Fisher 1986, Nattermann and Villain 1988); therefore

$$n \simeq \text{const}(H - H_{c1}) \qquad \partial M/\partial H|_{H\to H_{c1}+0} \simeq \text{const}.$$

An IC phase that arises at $H \geqslant H_{c1}$ has not even algebraic order. The phase diagram of such a junction is very interesting: it also contains (besides the Meissner and IC phases) a spin-glass phase (Kardar and Nelson 1985).

Equations (29) and (30) have a singularity at $\zeta = 1$ (random fields; Fisher 1986, Nattermann and Villain 1988). It probably means that the phase diagram contains the Meissner phase only for fields less than some $H_x \leqslant H_{c1}$. At $H = H_x$ the first-order phase transition into the normal state should take place. The mechanism of this transition is probably connected with the arising of rough Josephson vortices, which fill in all the surface of the junction.

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References

Aslamazov L G and Gurovich E V 1984 *Pis. Zh. Eksp. Teor. Fiz.* **40** 22
Bak P 1982 *Rep. Prog. Phys.* **45** 587
Barone A and Paterno G 1982 *Physics and Applications of the Josephson Effect* (Chichester: Wiley)
Browne D and Horovitz B 1988 *Phys. Rev. Lett.* **61** 1259
Dubson M A, Herbert S T, Calabrese J J, Harris D C, Patton B R and Garland J C 1988 *Phys. Rev. Lett.* **60** 1061
Fetter A L and Stephen M J 1968 *Phys. Rev.* **168** 475
Fisher M E 1984 *J. Stat. Phys.* **34** 667
—— 1986 *J. Chem. Soc. Faraday Trans. II* **82** 1569
Halperin B I, Lubensky T C and Ma S-K 1974 *Phys. Rev. Lett.* **32** 292
Kardar M and Nelson D R 1985 *Phys. Rev. Lett.* **55** 1157
Kolomeisky E B 1988 *Pis. Zh. Eksp. Teor. Fiz.* **48** 279 (Engl. Transl. *JETP Lett.* **48** 309)
Kolomeisky E B and Levanyuk A P 1989 *Pis. Zh. Eksp. Teor. Fiz.* **50** 45 (Engl. Transl. *JETP Lett.* **50** 288)
Kosterlitz J M and Thouless D J 1973 *J. Phys. C: Solid State Phys.* **6** 1181
Kulik I O 1966 *Pis. Zh. Eksp. Teor. Fiz.* **51** 1952

Kulik I O and Yanson I K 1970 *Effekt Dzhozefcona v Sverhprovodyaschikh Tunnel'nykh Strukturakh* (Moscow: Nauka)
Landau L D and Lifshitz E M 1976 *Statisticheskaya Fizika* I (Moscow: Nauka)
Lifshitz E M and Pitaevsky L P 1978 *Statisticheskaya Fizika* II (Moscow: Nauka)
Lyuksyutov I F and Pokrovsky V L 1982 *J. Physique Lett.* **43** L11
Nattermann T and Villain J 1988 *Phase Transitions* **11** 5
Nozieres P and Gallet F 1987 *J. Physique* **48** 353
Owen T and Scalapino D J 1967 *Phys. Rev.* **164** 538
Pokrovsky V L and Talapov A L 1979 *Phys. Rev. Lett.* **42** 65
Stamp P C E, Forro L and Ayache C 1988 *Phys. Rev.* B **38** 2847
Uimin G V and Pokrovsky V L 1984 *J. Stat. Phys.* **34** 775
Villain J 1980 *Ordering in Strongly Fluctuating Condensed Matter Systems* ed T Riste (New York: Plenum) p 221